Newtonian Mechanics

1. A particle moves with constant acceleration $A$ in the x-direction along the line $y = h$ having started from rest at $x = 0$. Write down the position, velocity, and acceleration of the particle in polar coordinates.

2. A hiker kicks a rock horizontally off the edge of a cliff, and it falls into a body of water below. If the hiker hears the sound of the splash $T$ seconds after having kicked the rock, and the height of the cliff above the water is $h$, what was the initial speed of the rock after the hiker kicked it? Assume the speed of sound is $v$.

3. A boy stands at the top of a hill which slopes downward uniformly at an angle $\phi$ below the horizontal. At what angle $\theta$ above the horizontal should the boy throw a tennis ball so that it has the greatest range?

4. Determine the force $F$ that must be applied to the large block with mass $m_1$ shown in the picture below in order to prevent the block with mass $m_3$ from moving up or down. All surfaces are frictionless and the pulley has negligible mass.

5. A block of mass $m$ rests on a wedge inclined at an angle $\theta$. The coefficient of static friction between the block and the wedge is $\mu_s$. The wedge is given a horizontal acceleration $a$ as shown in the figure below. Assuming that $\tan \theta > \mu_s$, determine the possible values of $a$ for which the block remains on the wedge without sliding up or down.
6. A large vat of sand steadily drains through a nozzle at its base. The stream of sand falls to the ground below and steadily accumulates in a cone shaped pile. If the coefficient of static friction between sand grains is $\mu_s$, how tall is the pile after a volume of sand $V$ has drained from the vat?

7. A bucket of water is suspended from a rope and rotated with angular velocity $\omega$ around its central axis which points along the direction of the rope. Determine the height of the water level inside the bucket as a function of the distance from the center of the bucket.

8. A circular ring of radius $R$ is suspended perfectly horizontally above the ground. A bead of mass $m$ is constrained to move around the ring and does so with an initial speed $v_0$ and begins at an initial angle $\theta_0$. The coefficient of kinetic friction between the bead and the ring is $\mu_k$. Determine the position of the bead as a function of time.

9. A particle of mass $m$ moving in a straight line with an initial speed $v_0 > 0$ is slowed by a resistive force whose magnitude is given by $F = bv^\alpha$, where $v$ is the speed of the particle. Determine the time required for the particle to come to rest.

10. A block of mass $m$ is placed on a ramp inclined at an angle $\theta$. Starting from rest, the block begins to slide down the ramp under the influence of gravity. As the block slides, it experiences a resistive force given by $F = mkv^2$, where $k$ is a positive constant, and $v$ is the block’s speed. Determine the amount of time required for the block to slide down a distance $d$.

11. A particle confined within the x-y plane moves along a trajectory with $r = a\theta$, where $a$ is a constant. The particle moves in such a way that the radial acceleration is always zero. If the particle starts with $\theta_0 = 0$ and an initial angular velocity $\omega_0$, determine the amount of time before the angle reaches $\theta$. Express your answer in terms of the Error function, $\text{erf}(x)$, defined by:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$