Springs and Oscillations

1. A block of mass $M$ rests on a horizontal, frictionless surface and is attached to a spring connected to a wall as shown in the figure below. A bullet of mass $m_b$ is fired with speed $v_0$ into the block and becomes lodged inside. This results in the block and spring undergoing simple harmonic motion. If the amplitude of oscillation is $\Delta x$, what is the period of oscillation?

2. A mass $m$ constrained to move along the x-axis is attached to an elastic cable with spring constant $k$. The other end of the cable is fixed at the origin, and there is no friction. The cable has negligible unstretched length, and the mass is initially sitting at the origin at rest. A constant force $F$ directed in the positive x-direction is then applied to the mass. Determine the position and velocity of the mass as functions of time.

3. A simple pendulum of length $\ell$ is raised to an angle $\phi$ and released from rest. The initial angle $\phi$ is not necessarily small. Determine the period of oscillation. You may express your final answer in terms of an integral (you won’t be able to evaluate it).

4. An underdamped oscillator with initial energy $E_0$ has a natural frequency $\omega_0$, damping constant $\beta$, and an initial phase angle $\phi$. Determine the rate at which the oscillator loses energy over time.

5. Determine the driving frequency at which the steady-state amplitude of a lightly damped ($\beta \ll \omega_0$) oscillator is half the amplitude at the resonant frequency.