Short Answer  
(5 Points Each)

1. Three electrons and one proton are contained inside a closed, cubic box with sides of length \( a = 4.20 \text{ cm} \). What is the electric flux through the box? The charge of an electron is \( Q_e = -1.60 \times 10^{-19} \text{ C} \).

\[
\Phi_E = \frac{Q_{\text{ins.}}}{\varepsilon_0} \implies \Phi_E = \frac{3Q_e - Q_e}{\varepsilon_0}
\]

\[
\implies \Phi_E = -3.61 \times 10^{-8} \text{ N m}^2 \text{ C}^{-1}
\]

2. Determine the angle between the following vectors:

\[
\vec{A} = 2\hat{i} + 6\hat{j} - 4\hat{k} \quad \text{and} \quad \vec{B} = -\hat{i} - 3\hat{j} + 4\hat{k}.
\]

\[
\hat{A} \cdot \hat{B} = \vec{A} \cdot \vec{B} = AB \cos(\theta) \implies -3 \cos(\theta) = (3, 8, 16) \cos(\theta)
\]

\[
\implies \theta = 160.6^\circ
\]

3. If the electric field is given by \( \vec{E} = \beta \left[ y^2 \hat{i} + (2xy + z^2) \hat{j} + 2yz \hat{k} \right] \) in some region of space, determine the charge density \( \rho(x, y, z) \) in that region. Treat \( \beta \) as a known constant.

\[
\rho = \varepsilon_0 (\nabla \cdot \vec{E}) \quad \nabla \cdot \vec{E} = \frac{\partial}{\partial x} \left[ \beta y^2 \right] + \frac{\partial}{\partial y} \left[ \beta (2xy + z^2) \right] + \frac{\partial}{\partial z} \left[ \beta 2yz \right]
\]

\[
\implies \rho = \beta \varepsilon_0 (2x + 2y) \quad \implies \rho = 2\beta \varepsilon_0 (x + y)
\]
4. Calculate the curl of the following vector function:

\[ \mathbf{F}(x, y, z) = xy \hat{i} + yz \hat{j} + zx \hat{k}. \]

\[ \nabla \times \mathbf{F} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
x & y & z
\end{vmatrix} = -y \hat{i} - z \hat{j} - x \hat{k} \]

5. Suppose the electric potential \( V \) in some region of space is given by the equation below for some known constant \( \mu \). What is the electric field in this region of space?

\[ V(x, y, z) = \frac{\mu}{xyz} \]

\[ \mathbf{E} = -\nabla V \Rightarrow \mathbf{E} = -\mu \left( \frac{\partial}{\partial x} (xyz)^{-1} \hat{i} - \frac{\partial}{\partial y} (xyz)^{-1} \hat{j} - \frac{\partial}{\partial z} (xyz)^{-1} \hat{k} \right) \]

\[ \Rightarrow \mathbf{E} = \mu \left[ \frac{1}{x^2yz} \hat{i} + \frac{1}{xyz^2} \hat{j} + \frac{1}{xyz^2} \hat{k} \right] \]

6. A droplet of ink in an ink-jet printer carries a charge of \( 4.50 \times 10^{-13} \) C and is deflected onto paper by a force of \( 1.80 \times 10^{-7} \) N. What was the strength of the electric field that produced this force?

\[ \mathbf{F} = q \mathbf{E} \Rightarrow (1.80 \times 10^{-7}) = E (4.50 \times 10^{-13}) \]

\[ \Rightarrow E = 4.00 \times 10^5 \text{ N/C} \]
7. Sketch the electric field diagram for a pair of oppositely charged particles. Both particles have the same magnitude of charge, but one is positive while the other is negative.

8. Three charged particles are arranged at the vertices of a right triangle. The first particle has charge $q$ and is located at the point $(3a, 0)$, the second particle has charge $2q$ and is located at $(0, 0)$, and the third particle has charge $3q$ and is located at $(0, 4a)$. How much electrical potential energy is stored in this configuration?

\[ W = W_1 + W_2 + W_3 \]
\[ W = \frac{1}{\epsilon_0} \left[ \frac{2q^2}{3a} + \frac{3q^2}{5a} + \frac{6q^2}{a} \right] \]
\[ = \frac{83}{30} \left( \frac{kq^2}{a} \right) \]
Problems  
(20 Points Each)

1. The following questions refer to the circuit diagram below. 1) Find the total effective resistance of the circuit. 2) Determine the voltage drop across resistors \(A\), \(B\), and \(C\). Give your answers to three significant figures.

\[
R_{\text{Total}} = 4_Ω + 1.875_Ω + R_{\text{Box}} + 8_Ω
\]

\[
\Rightarrow R_{\text{Total}} = 16.952_Ω \quad \Rightarrow \quad R_{\text{Total}} = 17.0Ω
\]

\[
\Delta V_A = \left(\frac{40V}{R_{\text{Total}}}\right)(4_Ω) = 9.438V \quad \rightarrow \quad \Delta V_A = 9.44V
\]

\[
\Delta V_{\text{Top Box}} = \left(\frac{40V}{R_{\text{Total}}}\right)(1.875_Ω) = 4.424V = \Delta V_B \rightarrow \Delta V_B = 4.42V
\]

\[
\Delta V_{\text{Box}} = \left(\frac{40V}{R_{\text{Total}}}\right)R_{\text{Box}} = 7.2605V \quad \Rightarrow \quad I_{\text{Bottom}} = \frac{7.2605V}{8_Ω} = 0.9076A
\]

\[
\Rightarrow \Delta V_C = I_{\text{Bottom}} \cdot 6_Ω = 5.445V \quad \rightarrow \quad \Delta V_C = 5.45V
\]
2. A uniformly charged rod of length \( \ell \) lies along the \( x \)-axis with its left end at the origin. Determine the electric field everywhere. (In other words, determine the \( x \) and \( y \) components of the electric field as functions of the coordinates \( x \) and \( y \).)

\[
\begin{align*}
\begin{array}{c}
\text{Let } \theta = \angle \text{ between } \overrightarrow{OE} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\frac{dE_x}{dE} &= \cos(\theta) \frac{dE}{dE} \\
\frac{dE_y}{dE} &= \sin(\theta) \frac{dE}{dE} \\
\frac{dE}{dE} &= K \frac{d\theta}{\ell^2} \\
\end{align*}
\]

\[
\begin{align*}
\theta &= \frac{x}{\ell^2} \\
\end{align*}
\]

\[
\begin{align*}
\cos(\theta) &= \frac{x-x'}{\ell} \\
\sin(\theta) &= \frac{y}{\ell} \\
\end{align*}
\]

\[
\begin{align*}
d\theta &= \lambda \frac{dx'}{\ell} \\
\lambda &= \frac{Q}{\ell} \\
\end{align*}
\]

\[
\begin{align*}
E_x &= \int_{0}^{\ell} \frac{\lambda \, dx'}{\ell^2} \\
&= \int_{0}^{\ell} \cos(\theta) \frac{dE}{dE} \\
&= \int_{0}^{\ell} \frac{(x-x') \, dx'}{[(x-x')^2 + y^2]^{3/2}} \\
\end{align*}
\]

\[
\begin{align*}
E_y &= \int_{0}^{\ell} \frac{\lambda \, dx'}{\ell^2} \\
&= \int_{0}^{\ell} \sin(\theta) \frac{dE}{dE} \\
&= \int_{0}^{\ell} \frac{y \, dx'}{[(x-x')^2 + y^2]^{3/2}} \\
\end{align*}
\]

\[
\text{Let's evaluate } E_x 	ext{ first...}
\]

\[
\begin{align*}
\text{Let } u &= (x-x')^2 + y^2 \\
\Rightarrow \quad du &= -2(x-x') \, dx' \\
\end{align*}
\]

\[
\begin{align*}
\text{Hence,} \\
E_x &= \frac{KQ}{\ell} \int_{x'=0}^{x' = \ell} \frac{-du}{2u^{3/2}} \\
&= \frac{KQ}{\ell} \left[ \frac{1}{u^{1/2}} \right]_{x^2+y^2}^{(x-x')^2+y^2} \\
&= \frac{KQ}{\ell} \left[ \frac{1}{\sqrt{x^2+y^2}} - \frac{1}{\sqrt{(x-x')^2+y^2}} \right] \\
\end{align*}
\]
Now let's evaluate $E_y...$

$$E_y = \frac{KQ}{L} y \int_0^L \frac{dx'}{[(x-x')^2+y^2]^{3/2}} = \frac{KQ}{L} y \int_0^L \left[ y^2 \left(\frac{x-x'}{y} \right)^2 + 1 \right]^{-3/2} dx'$$

$$= \frac{KQ}{L} y \int_0^L y^3 \left(\frac{x-x'}{y} \right)^2 + 1 \right]^{-3/2} dx' = \frac{KQ}{L y^2} \int_0^L \left[ \left(\frac{x-x'}{y} \right)^2 + 1 \right]^{-3/2} dx'$$

Let $\left(\frac{x-x'}{y} \right) = \tan(\theta) \Rightarrow \frac{-1}{y} dx' = \sec^2(\theta) d\theta$

$$\Rightarrow E_y = \frac{KQ}{L y^2} \int_{\theta=0}^{\theta=L} -y \sec^2(\theta) d\theta = \frac{-KQ}{L y} \int_{\theta=0}^{\theta=L} \frac{\sec^2(\theta) d\theta}{\sec^3(\theta)}$$

$$= \frac{-KQ}{L y} \int_{\theta=0}^{\theta=L} \cos(\theta) d\theta = \frac{-KQ}{L y} \left[ \sin(\theta) \right]_{\theta=0}^{\theta=L}$$

$$= \frac{-KQ}{L y} \left[ \frac{x-L}{\sqrt{(x-L)^2+y^2}} - \frac{x}{\sqrt{x^2+y^2}} \right]$$

Thus:

$$E = E_x \mathbf{\hat{i}} + E_y \mathbf{\hat{j}}$$
3. A very long, solid cylinder has a charge density that is greater farther from its central axis so that \( \rho = ar \), where \( r \) is the radial distance from the cylinder's central axis, and \( a \) is a constant. What is the electric field inside the cylinder?

\[
\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{INS.}}}{\varepsilon_0}
\]

**First Note:** \( \int E_{\text{top}} \cdot dA_{\text{top}} = \int E_{\text{bottom}} \cdot dA_{\text{bottom}} = 0 \) since \( E \) and \( d\vec{A} \) are perpendicular at the top & bottom.

**Next**

\[
\int_{\text{side}} E \cdot dA = \int E dA \cos(\theta) = E \int dA = E (2\pi rh)
\]

\[
Q_{\text{INS.}} = \int S \, dv = \int_0^r \int_0^{2\pi} \int_0^h (ar) \, rd\theta \, dz \\
\Rightarrow Q_{\text{INS.}} = 2\pi r a \int_0^r \int_0^{2\pi} z \, dz = \frac{2\pi r a}{3} r^3
\]

Hence

\[
E (2\pi rh) = \frac{2\pi r a}{3\varepsilon_0} r^3
\]

\[
\Rightarrow E = \frac{a}{3\varepsilon_0} r^2
\]

The direction points radially outwards!