Short Answer
(5 Points Each)

1. Calculate the curl of the following vector function and state whether it is a possible electrostatic field.

\[ \mathbf{E}(x, y, z) = x^2 y \mathbf{i} + y^2 z \mathbf{j} + z^2 x \mathbf{k} \]

\[ \nabla \times \mathbf{E} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
x^2 y & y^2 z & z^2 x
\end{vmatrix} = -y^2 \mathbf{\hat{i}} - z^2 \mathbf{\hat{j}} - x^2 \mathbf{\hat{k}} \]

\[ \nabla \times \mathbf{E} = -y^2 \mathbf{\hat{i}} - z^2 \mathbf{\hat{j}} - x^2 \mathbf{\hat{k}} \]

\[ \boxed{\mathbf{\nabla \times \mathbf{E} = -y^2 \mathbf{\hat{i}} - z^2 \mathbf{\hat{j}} - x^2 \mathbf{\hat{k}}} \]

2. The decay rate for a group of \( 8.20 \times 10^{18} \) radioactive nuclei (all the same isotope) is 578 Ci. What is the half life of this isotope?

\[ T_{1/2} = \frac{\ln(2)}{\lambda} \]

\[ \ln(N(t)) - \ln(N_0) = -\lambda t \]

\[ N = \frac{N_0}{2} \]

\[ R = 8.20 \times 10^{18} \]

\[ N = 578 \text{ Ci} \]

\[ R = 578 \text{ Ci} = 2.14 \times 10^{13} \text{ decays per second} \]

\[ \boxed{T_{1/2} = 3.0 \text{ years}} \]

3. A man standing 1.54 meters in front of a shaving mirror produces an inverted image 21.2 cm in front of it. How close to the mirror should he stand if he wants to form an upright image of his chin that is twice the chin’s actual size?

\[ P_1 = 154 \text{ cm} \]

\[ \frac{1}{P_1} + \frac{1}{b_1} = \frac{1}{f} \]

\[ f = \frac{18.6 \text{ cm}}{2} \]

\[ b_1 = 21.2 \text{ cm} \]

\[ M = +2 \]

\[ 2 = \frac{b}{P} \]

\[ \Rightarrow 2 = \frac{21.2}{P} \]

\[ \Rightarrow P = 9.3 \text{ cm} \]

\[ \Rightarrow \frac{1}{P} + \frac{1}{(-2P)} = \frac{1}{18.6 \text{ cm}} \]

\[ \Rightarrow P = 9.3 \text{ cm} \]
4. A proton \((m_p = 1.67 \times 10^{-27} \text{ kg})\) is determined to be located inside a spherical cavity of diameter \(3.20 \times 10^{-6} \text{ meters}\). What is the minimum uncertainty in its speed?

\[
\sigma_x \sigma_p \geq \frac{\hbar}{4\pi} \quad \sigma_x = \frac{1}{2} (3.20 \times 10^{-6}) \quad \sigma_p = M \sigma_v
\]

\[
\Rightarrow \quad \frac{1}{2} (3.20 \times 10^{-6}) (1.67 \times 10^{-27}) \sigma_v \geq \frac{1}{4\pi} (6.63 \times 10^{-34})
\]

\[
\text{Hence} \quad \sigma_v \geq 1.97 \times 10^{-2} \text{m/s}
\]

5. Light traveling through the air hits a slab of translucent material at an angle of \(40.0^\circ\). What is the speed of light in the material if the angle of refraction in the material is \(28.0^\circ\)?

\[
N_{\text{air}} \sin(\theta_a) = N \sin(\theta_r) \quad \Rightarrow \quad N = \frac{\sin(40.0^\circ)}{\sin(28.0^\circ)} = 1.37
\]

Now use \(N = \frac{c}{v}\)

\[
\text{Hence} \quad v = 2.19 \times 10^8 \text{m/s}
\]

6. Two electric circuits are constructed. Each circuit has a total of \(N\) resistors, and each resistor has resistance \(R\). The first circuit has all of the resistors in series. The second has all of them in parallel. If both circuits have a power source operating with voltage \(V\), determine the ratio of their currents.

\[
R_s = R + R + \ldots + R = NR \quad \Rightarrow \quad I_s = \frac{V}{NR}
\]

\[
R_p = \left[ \frac{1}{R} + \frac{1}{R} + \ldots + \frac{1}{R} \right]^{-1} = \frac{R}{N} \quad \Rightarrow \quad I_p = \frac{NV}{R}
\]

\[
\text{Hence} \quad \frac{I_p}{I_s} = N^2
\]
7. Two charged particles, each with charge $q$ are fixed in place on the x-axis and are separated by a distance $a$. The left particle is located at the origin. Suppose a third particle, also having charge $q$, is placed at the point $(x, y)$. Determine the total electric potential energy stored in the resulting configuration.

\[ W_1 = 0 \quad V_1 = \sqrt{x^2 + y^2} \]
\[ W_2 = \frac{kq^2}{a} \quad V_2 = \sqrt{(x-a)^2 + y^2} \]
\[ W_3 = \frac{kq^2}{r_1} + \frac{kq^2}{r_2} \]

\[ \Rightarrow \quad W = kq^2 \left[ \frac{1}{a} + \frac{1}{\sqrt{x^2 + y^2}} + \frac{1}{\sqrt{(x-a)^2 + y^2}} \right] \]

8. A long vertical wire lies in the plane of this page with 3.50 A of current flowing upwards through it. A second wire also lies in the plane of this page, but it stretches horizontally and has 2.80 A of current flowing to the right. Determine the magnitude and direction of the magnetic field at the points 6.80 cm from the vertical wire and 4.20 cm from the horizontal wire.

\[ B_{\text{vertical}} = \frac{\mu_0 I_v}{2\pi R_v} = 1.03 \times 10^{-5} \ T \]
\[ B_{\text{horizontal}} = \frac{\mu_0 I_h}{2\pi R_h} = 1.33 \times 10^{-5} \ T \]

\[ B_A = 3.04 \times 10^{-6} \ \text{out of page!} \]
\[ B_B = 2.36 \times 10^{-5} \ \text{out of page!} \]
\[ B_C = 3.04 \times 10^{-6} \ \text{into page!} \]
\[ B_D = 2.36 \times 10^{-5} \ \text{into page!} \]

**Note:** $(B_{\text{vertical}}, B_{\text{horizontal}})$
Problems
(20 Points Each)

1. Suppose a radioactive substance has a half life of 30.0 days. If there are initially $4.00 \times 10^{18}$ nuclei of this substance, what is the activity four years later? Express your answer in Curies.

$$N(t) = N_0 e^{-\lambda t} \quad R = \lambda N \quad \lambda = \frac{\ln(2)}{T_{1/2}}$$

$$\Rightarrow \quad \lambda = 2.67 \times 10^{-7} \quad \text{and} \quad t = 1.26 \times 10^8$$

Hence

$$R(t) = \lambda N_0 e^{-\lambda t}$$

$$= \left(2.67 \times 10^{-7}\right) \left(4.00 \times 10^{18}\right) e^{-(2.67 \times 10^{-7})(1.26 \times 10^8)}$$

$$= 2.34 \times 10^3 \text{ Bq}$$

Now convert to Curies!

$$\Rightarrow \quad R = 6.32 \times 10^{14} \text{ Ci}$$
2. A half circle of radius $R$ has total charge $Q$, but the charge is not uniformly distributed. Instead, the charge density at a point an angle $\theta$ counterclockwise from the vertical is proportional to the absolute value of the cosine of that angle. In other words, $\lambda(\theta) \propto |\cos(\theta)|$. Determine the value of the electric field at the center of the half circle.

\[ d\mathbf{E} = \lambda \mathbf{d}s = \lambda R d\theta = a R |\cos(\theta)| \, d\theta \]

First find $a$...

\[ Q = \int_0^{\pi} \lambda(\theta) R d\theta \]

\[ dE_x = dE \sin(\theta) \quad \Rightarrow \quad Q = \int_0^{\pi} a R |\cos(\theta)| \, d\theta = 2a R \int_0^{\pi/2} \cos(\theta) \, d\theta = 1 \]

Hence $a = \frac{Q}{2R}$ so...

\[ \lambda = \frac{Q}{2R} |\cos(\theta)| \]

\[ dE_x = K \frac{d\theta}{R^2} \sin(\theta) \quad \text{Plug this in for } d\mathbf{E}... \]

\[ \Rightarrow \quad dE_x = \frac{K}{R^2} \left( \frac{Q}{2} \right) |\cos(\theta)| \sin(\theta) \, d\theta \]

\[ \Rightarrow \quad E_x = \frac{KQ}{2R^2} \int_0^{\pi/2} |\cos(\theta)| \sin(\theta) \, d\theta = \frac{KQ}{R^2} \int_0^{\pi/2} \cos(\theta) \sin(\theta) \, d\theta \]

\[ = \frac{KQ}{2R^2} \int_0^{\pi/2} \sin(\theta) \, d\theta = \Rightarrow \quad E_x = \frac{KQ}{2R^2} \quad \text{Note that the } y\text{-components cancel, so...} \]

\[ E_x = \frac{KQ}{2R^2} \quad \text{E}_y = 0 \text{ and} \]

\[ E = \frac{KQ}{2R^2} \hat{i} \]
3. People on Earth see a spacecraft moving away from Earth at half the speed of light. It launches a daughter ship in the same direction (away from Earth) and the daughter ship moves at half the speed of light relative to the mothership. The daughter ship then fires a rocket in the same direction (also away from Earth). The rocket moves at half the speed of light relative to the daughter ship. If the length of the rocket is 3.00 meters in its rest frame, what is its length as seen from Earth?

\[ V_{AE} = V_{BA} = V_{CB} = (0.500) \, C \]

\[ L_o = 3.00 \, \text{m} \]

\[ L = L_o \sqrt{1 - \frac{V_{CE}^2}{C^2}} \]

**First we need** \( V_{CE} \),

**Step 1:**

\[ V_{CA} = \frac{V_{CB} + V_{BA}}{1 + \frac{V_{CA} \cdot V_{BA}}{C^2}} = \frac{C}{1 + (0.250)} = (0.800) \, C \]

**Step 2:**

\[ V_{CE} = \frac{V_{CA} + V_{AE}}{1 + \frac{V_{CA} \cdot V_{AE}}{C^2}} = \frac{(1.30) \, C}{1 + (0.400)} = (0.929) \, C \]

**Step 3:**

\[ L = (3.00 \, \text{m}) \sqrt{1 - (0.929)^2} \]

\[ \Rightarrow \quad L = 1.11 \, \text{m} \]