Basic Vector Calculus

1. Calculate the following determinants:

\[
\begin{vmatrix}
6 & 2 & 4 \\
5 & 2 & 1 \\
3 & 6 & 3 \\
\end{vmatrix}
= ?
\]
\[
\begin{vmatrix}
4 & -3 & 5 \\
2 & 1 & 5 \\
-3 & 8 & 6 \\
\end{vmatrix}
= ?
\]

For the following two problems, let \( \vec{A} = 3 \hat{i} - 5 \hat{j} + 8 \hat{k} \) and \( \vec{B} = -4 \hat{i} + 3 \hat{j} - 2 \hat{k} \).

2. Determine the angle between vectors \( \vec{A} \) and \( \vec{B} \).

3. Calculate the cross product: \( \vec{A} \times \vec{B} = ? \)

4. Calculate the gradient of the following scalar functions:
   a) \( f(x, y, z) = xy^2 z^3 \)
   b) \( r = \sqrt{x^2 + y^2 + z^2} \)

5. Calculate the divergence of the following vector functions:
   a) \( \vec{G}(x, y, z) = x \hat{i} + xy \hat{j} + z^3 \hat{k} \)
   b) \( \vec{H}(x, y, z) = 6y \hat{i} + 2xz \hat{j} + 4x^2 \hat{k} \)

6. Calculate the curl of the following vector functions:
   a) \( \vec{G}(x, y, z) = -y \hat{i} + x \hat{j} \)
   b) \( \vec{H}(x, y, z) = x \hat{j} \)

7. Sketch both of the vector fields from the previous problem. Do the results for the curls that you calculated for them make sense? Explain.
Given an arbitrary scalar function, \( f(x, y, z) \), the following identity is generally true:

\[
\frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right).
\]

Use this fact for the following problems.

8. Calculate the curl of the gradient of an arbitrary scalar function \( f(x, y, z) \). Show your work.

\[
\vec{\nabla} \times \vec{\nabla} f = ?
\]

9. Let \( \vec{F}(x, y, z) = F_x(x, y, z) \hat{i} + F_y(x, y, z) \hat{j} + F_z(x, y, z) \hat{k} \) be an arbitrary vector function. Calculate the divergence of the curl of \( \vec{F} \). Show your work.

\[
\vec{\nabla} \cdot \left( \vec{\nabla} \times \vec{F} \right) = ?
\]

10. Calculate the curl of the electric field caused by the presence of a stationary point charge \( q \).