Special Relativity

1. A tilted rod with proper length $\ell_0$ makes an angle $\theta_0$ with respect to the x-axis in its own rest frame. What is the length and orientation of the rod in the lab frame, in which the rod is moving to the right with speed $v$?

2. A solid cube of stone has sides of rest length $\ell_0$. Determine the volume of the same stone in a frame in which it is observed to be moving with speed $v$. Prove that your answer does not depend on the direction of the stone’s observed velocity relative to its orientation.

3. An inertial observer in frame $S$ sees two spaceships moving in opposite directions along straight, parallel trajectories that are separated by a distance $d$. Each ship moves with a constant speed $v = c/2$. At the moment when the two ships are closest (as seen in frame $S$), ship A launches a cargo vessel with speed $u = 3c/4$ (also as seen in frame $S$). From the perspective of those aboard ship A, with what speed and at what angle must the cargo vessel be launched in order for it to successfully intercept ship B?

4. Consider two inertial reference frames $S$ and $S'$. The axes of both frames are aligned, but the frame $S'$ moves with constant speed $v$ down the positive x-axis of the frame $S$. The origins of both frames coincide at $t = t' = 0$. In frame $S'$ a particle moves in a straight line along the x'-axis with acceleration $a'$ and instantaneous velocity $u'$. Determine the acceleration of the particle in the frame $S$.

5. A particle of rest mass $m_0$ and kinetic energy $km_0c^2$ strikes and sticks to an identical particle at rest ($k$ is some positive integer). What is the rest mass of the resultant particle?

6. A gamma ray collides with a proton at rest and produces a neutral pion according to the reaction: $\gamma + p \rightarrow p + \pi^0$. What is the minimum energy the gamma ray must have for this reaction to occur?

7. Determine the shift in wavelength of a photon that scatters elastically off of a charged particle of mass $m$ at a scattering angle $\theta$. 
8. Particle $A$ decays into several lighter particles: $A \rightarrow B_1 + B_2 + \ldots + B_n$ for $n \geq 3$. Determine the minimum and maximum possible energies that $B_1$ can have in terms of the particle masses: $M_A, m_1, m_2, \ldots, m_n$.

9. In a (relativistic) candy factory, a long sheet of chewy taffy moves along a conveyor belt with speed $v$. Circular cutters stamp down onto the taffy as it moves (with relativistic speed) beneath them, cutting out pieces to be packaged farther down along the conveyor belt. If you bought this taffy in a store, what shape would it be? Would the pieces be stretched, squashed, or circular? Explain.

10. A spacecraft leaves Earth along a straight trajectory with a constant proper acceleration $g$ (at each moment this is the ship’s acceleration as measured in an inertial frame that is instantaneously at rest with respect to the ship). The ship spends a period of time accelerating away from Earth before eventually reversing thrusters and then spending the same length of time decelerating until it is at rest again. It then turns around and repeats this process, returning to Earth. The entire trip takes a time $\tau$ for the passengers of the ship.

a) How much time will the passengers of the ship find has passed on Earth when they get home?

b) How far away from Earth did the passengers of the ship travel?

c) What are your answers to parts (a) and (b) if $\tau = 30$ years?

11. A particle moves along the $x$-axis in frame $S$ starting from rest. The particle maintains a constant proper acceleration $\alpha$.

a) Show that the particle’s equation of motion in frame $S$ is given by:

$$\alpha x^2 + 2c^2 x - \alpha c^2 t^2 = 0.$$ 

b) Prove that there exists a critical time $T$ after which light signals emitted from the origin will never reach the receding particle. Determine this time in terms of $\alpha$ and sketch a spacetime diagram illustrating the particle’s worldline along with that of the light signal.
12. With respect to the ground, Dylan moves with speed $v$ to the right while Kelly moves with speed $v$ to the left. The instant they are a distance $\Delta x$ apart (in the ground’s frame) Dylan claps his hand. Kelly then claps her hand simultaneously (as measured in her frame) with Dylan’s clap. Dylan then claps his hand simultaneously (as measured in his frame) with Kelly’s clap. Kelly then claps her hand simultaneously (as measured in her frame) with Dylan’s second clap, and so on. As measured in the ground frame, how far apart are Dylan and Kelly when Dylan makes his $n$th clap?