1. In the lab frame, a gamma ray with energy $E_\gamma$ is incident upon a particle $A$ with mass $M$ that is at rest. The gamma ray is absorbed, creating a new, unstable particle $B$. A detector is set up to register the decay products of $B$ so that scientists can determine how far it traveled before decaying.

a) Determine the mass of particle $B$. (15 Points)

b) If particle $B$ has a lifetime $\tau$ (in its own frame), determine the distance particle $B$ will have traveled in the lab frame before decaying. (15 Points)

\[
\begin{align*}
\text{a)} \quad & \gamma \quad \rightarrow \quad A \quad \rightarrow \quad B \quad \nu \quad \rightarrow \quad P_\gamma^\mu + P_A^\mu = P_B^\mu \\
\text{SQUARE BOTH SIDES,} \quad & \Rightarrow \quad 0 + 2P_\gamma \cdot P_A + M^2 = M_B^2 \\
\text{But} \quad & P_\gamma \cdot P_A = E_\gamma M \quad \Rightarrow \quad \boxed{M_B = M\sqrt{1 + \frac{2E_\gamma}{M}}} \\
\text{HENCE} \quad & \\
\text{b)} \quad & \Delta x_{\text{LAB}} = \nu \cdot z_{\text{LAB}} \quad \Rightarrow \quad z_{\text{LAB}} = \gamma z \quad \Rightarrow \quad \Delta x_{\text{LAB}} = \gamma \nu z \\
\text{NOW} \quad & P_B = \gamma M_B \nu \quad \Rightarrow \quad \gamma \nu = \frac{P_B}{M_B} = \frac{E_\gamma}{M_B} \quad \left( \text{p}_B = p_\gamma = E_\gamma \quad \text{G/C of} \quad \text{MOMENTUM CONSERVATION} \right) \\
\Rightarrow \quad & \Delta x_{\text{LAB}} = \frac{E_\gamma z}{M_B} \quad \Rightarrow \quad \boxed{\Delta x_{\text{LAB}} = \frac{E_\gamma z}{M\sqrt{1 + \frac{2E_\gamma}{M}}}}
\end{align*}
\]
2. Let \( p^\mu = (10, -3, 2, 4) \) and \( k^\mu = (6, 0, 5, 1) \) be four-vectors, and let \( g_{\mu\nu} \) be the Minkowski metric tensor. Evaluate the following. (5 Points Each)

a) \( g^{\mu\nu} k_\mu k_\nu \rightarrow 10 \)

b) \( k^\mu p^\nu p_\nu \rightarrow (426, 0, 355, 71) \)

c) \( g_{\mu\nu}(p^\mu + k^\mu)p^\nu \rightarrow 117 \)

d) \( g^{\mu\nu} k_\sigma (k_\rho + p_\rho)(k_\lambda - p_\lambda) g^{\rho\sigma} g^{\lambda\alpha} g_{\mu\alpha} k_\alpha \rightarrow -2016 \)

\[ p^2 = 100 - 9 - 4 - 16 = 71 \quad k^2 = 36 - 0 - 25 - 1 = 10 \]

\[ p \cdot k = 60 - 0 - 10 - 4 = 46 \quad \left( p \cdot k = k \cdot p \right) \]

a) \( g^{\nu\nu} k_\nu k_\nu = k^2 = 10 \)

b) \( k^\nu p^\nu p_\nu = k^\nu (p^2) = (426, 0, 355, 71) \)

c) \( g_{\nu\nu} (p^\nu + k^\nu) (p^\nu) = p^2 + p \cdot k = 71 + 46 = 117 \)

d) \( k^\lambda (k_\lambda + p_\lambda) (k_\lambda - p_\lambda) k^\lambda = (k^2 + p \cdot k)(k^2 - k \cdot p) \)

\[ = \left( 10 + 46 \right) \left( 10 - 46 \right) = -2016 \]
3. For each process, determine whether it is possible or impossible. If possible, state which interaction allows it. If impossible, state which conservation law prohibits it. (5 Points Each)

a) $\eta \rightarrow \gamma + \gamma$

Possible, E&M Interaction

b) $\Sigma^+ + n \rightarrow \Sigma^- + p$

Impossible, Violates Charge Conservation!

c) $\mu^- \rightarrow e^- + \bar{\nu}_e$

Impossible, Violates Muon Number Conservation!

d) $n \rightarrow p + e^- + \bar{\nu}_e$

Possible, Weak Interaction

e) $\Omega^- \rightarrow \Delta + \Xi^-$

Impossible, Violates Energy Conservation and Also Baryon Number Conservation!

f) $\Sigma^+ \rightarrow p + \pi^0$

Possible, Weak Interaction
4. A neutral pion can be created via the photoproduction reaction given below. If the initial proton is at rest in the lab frame, determine the minimum necessary energy of the incident gamma ray in order to trigger this reaction. Take the masses of the particles as given. (20 Points)

\[ \gamma + p \rightarrow p + \pi^0 \]

\[ P_{\pi}^\nu = (E_\pi + M_\pi, 0, 0, 0) \quad P_{\pi}^\nu = (M_\pi + M_\pi, 0, 0, 0) \]

\[ (P_{\text{Total}})^2 = (P_{\text{Total}})^2 \]

\[ E_\gamma + M_p = (M_p + M_\pi)^2 \]

\[ E_\gamma^2 + 2E_\gamma M_p + M_p^2 = M_\pi^2 + 2M_p M_\pi + M_\pi^2 \]

\[ E_\gamma = \frac{M_\pi (M_\pi + 2M_p)}{2M_p} \]
★ Bonus! ★ (5 Points)

On the first homework, you showed that a light signal sent after a particle accelerating away from the origin with a constant proper acceleration \( a \) will never reach the particle so long as the particle has a head start that is at least \( T = c/\alpha \).

Imagine this scenario from the perspective of the particle. How can the light fail to catch up? Or does it? Explain.

**THE SPEED OF LIGHT IS CONSTANT FOR ALL INERTIAL OBSERVERS. THE PARTICLE IS ACCELERATING AND IS THEREFORE NOT IN AN INERTIAL FRAME! HENCE THE LIGHT SIGNAL IS NOT MOVING WITH SPEED C IN THIS FRAME AND WILL NOT CATCH UP!**