Review Problems

1. A particle of mass $m$ in an infinite square well of length $a$ has an initial wavefunction $\Psi(x, 0) = A \sin^5(\pi x / a)$.
   a) What is the normalized wavefunction $\Psi(x, t)$ at a later time $t$?
   b) What are the possible results of an energy measurement and their corresponding probabilities?
   c) What is the expectation value of the particle’s energy?
   d) At time $t$, what is the probability of finding the particle in the right side of the box (i.e. in the region $a/2 < x < a$)?

2. Calculate the uncertainty product $\sigma_x \sigma_p$ for the $n$th stationary state of the infinite square well.

3. Show that for every normalizable solution to the Schrödinger equation $\psi$, the corresponding energy $E$ must be greater than the global minimum of the potential $V(x)$. (See Griffiths problem 2.2 for a hint.)

Bound State Systems

4. A particle of mass $m$ is confined within the potential

   $$V(x) = \begin{cases} 
   \infty, & x < 0 \\
   0, & 0 \leq x \leq a \\
   V_0, & a < x 
   \end{cases}$$

   a) Determine the bound state solutions to the Schrödinger equation.
   b) Derive an equation for the bound state energies (you do not need to explicitly solve it).
   c) Sketch a qualitative plot of the ground state wavefunction without using any equations or a computer.
5. A particle of mass $m$ is subject to the following potential

$$V(x) = -\lambda [\delta(x - a) + \delta(x + a)].$$

Determine the ground state energy eigenfunction and obtain an equation for its corresponding energy.

6. A particle of mass $m$ moves under the influence of a delta function well located a distance $d$ from a hard wall. The potential is given by

$$V(x) = \begin{cases} 
\infty, & x < -d \\
-V_0 \delta(x), & x > -d.
\end{cases}$$

a) Under what conditions will there be a bound state solution?
b) Under what conditions can the presence of the hard wall be neglected?
c) Determine an approximate expression for the bound state energy if the wall is far from the well.

**Transmission & Reflection**

7. Suppose a wave comes in from the left and is incident upon a finite potential barrier of height $V_0$ and width $2a$:

$$V(x) = \begin{cases} 
0, & x < -a \\
V_0, & -a \leq x \leq a \\
0, & a < x.
\end{cases}$$

a) Calculate the transmission coefficient for the case $E < V_0$.
b) Calculate the transmission coefficient for the case $E = V_0$.
c) Calculate the transmission coefficient for the case $E > V_0$. 

8. A beam of particles with energy $E$ is incident from the left upon a finite potential step:

$$V(x) = \begin{cases} 
0, & x \leq 0 \\
V_0, & x > 0.
\end{cases}$$

a) What fraction of the beam is reflected back if $E < V_0$?

b) What fraction of the beam is reflected back if $E > V_0$?

9. An electron of energy $E = 1$ eV is incident upon a rectangular potential barrier of height $V_0 = 2$ eV. How wide must the barrier be in order for the transmission probability to be $\sim 10^{-3}$?