Quantum Mechanics in Three Dimensions

1. Suppose the kinetic energy of a neutron, confined inside a cubic box, is 10 MeV in its ground state. Calculate the size of the box and determine the energies of the next three excited states.

2. Work out the specific functional forms of $Y_{3}^{2}(\theta, \phi)$ and $Y_{2}^{-1}(\theta, \phi)$ using the general equation for spherical harmonics and the associated Legendre functions (from class notes or Equations 4.27, 4.28, and 4.32 in the Griffiths).

3. A particle of mass $m$ is located inside a finite spherical well. The potential is given by

$$V(x) = \begin{cases} -V_0, & r \leq a \\ 0, & r > a \end{cases}.$$  

Solve the radial equation with $\ell = 0$ to find the ground state and obtain the transcendental equation that determines whether there is a bound state.

4. An electron is trapped inside a hard, spherical cavity of radius $R$. Assume the cavity walls are impenetrable. If the electron is in the ground state, determine the pressure it exerts on the cavity walls. (Hint: recall problem #6 from homework #2.)

The Hydrogen Atom

(Note: For all problems that follow, the constant $a$ is the Bohr radius.)

5. Determine the probability of locating a ground state electron between $R_1 = a/2$ and $R_2 = 3a/2$ in a hydrogen atom.

6. What is the most likely distance from the nucleus to find a ground state electron in a hydrogen atom?
7. What is the most likely distance from the nucleus to find an electron in the $2p$ state of a hydrogen atom? (Being in the $2p$ state means that $n = 2$ and $\ell = 1$.)

8. What is the probability of finding the electron of a $1s$ hydrogen atom farther than $\beta a$ from the nucleus for an arbitrary constant $\beta$?

9. Calculate $\sigma_r$ and $\sigma_x$ for an electron in the ground state of hydrogen.

10. Calculate $\langle x^2 \rangle$ for an electron in the $n = 2$, $\ell = 1$, $m = 1$ state of hydrogen. (See Griffiths problem 4.13 for a hint.)