Idenical Particles

1. Two noninteracting particles of mass \( m \) reside in an infinite square well. Suppose one is in the \( n \)th stationary state and the other is in the \( m \)th stationary state and assume \( n \neq m \).
   a) Calculate \( \langle (x_1 - x_2)^2 \rangle \) if the two particles are distinguishable.
   b) Calculate \( \langle (x_1 - x_2)^2 \rangle \) if the two particles are identical bosons.
   c) Calculate \( \langle (x_1 - x_2)^2 \rangle \) if the two particles are identical fermions.

2. Two particles of mass \( m \) are stuck in a one dimensional, quantum harmonic oscillator with spring constant \( k \). The particles also interact with one another via an interaction potential given by \( V_{\text{int}}(x_1, x_2) = \frac{\lambda}{2}(x_1 - x_2)^2 \).
   a) Determine the energy spectrum. (Hint: make a change of variables in the Hamiltonian.)
   b) What are the three lowest energy levels if the two particles are identical spin zero bosons?
   c) What are the three lowest energy levels if the two particles are identical spin 1/2 fermions?
   d) What is the total spin for each of the three states from part (c)?

Addition of Angular Momentum

3. A system is composed of two particles. Particle A has spin 1, and particle B has spin 2.
   a) What are the possible values of the \( z \)-component of each of the two particles, and what are the possible values of the total spin of the system?
   b) If the system is in a configuration in which the total spin is 3 and the \( z \)-component of the total spin is 1, then what are the possible values and corresponding probabilities that one could measure for the \( z \)-component of the spin of particle B?
Higher Spin States

4. In class we worked out the matrices representing $S_x$, $S_y$, and $S_z$ for spin 1/2 systems. For this problem, you will determine the matrices representing $S_x$, $S_y$, and $S_z$ for higher spin states. For spin 1, there are three possible $m_s$ values, each corresponding to a column vector in the standard basis of $\mathbb{R}^3$. Recalling the fact that it is conventional to work in the $S_z$ basis, start with the fact that $S_\pm = S_x \pm i S_y$ and derive the $3 \times 3$ matrices $S_x$, $S_y$, and $S_z$ for spin 1 systems. You may use the fact that

$$S_\pm |s, m_s\rangle = \sqrt{(s \mp m_s)(s \pm m_s + 1)} |s, m_s \pm 1\rangle.$$ 

5. Now apply the same procedure to determine the matrices representing $S_x$, $S_y$, and $S_z$ for spin 3/2 systems.

Perturbation Theory

6. The proton is not a point charge, but instead has a charge distribution with a radius $R \approx 8.78 \times 10^{-16}$ m.

a) Using first order perturbation theory, calculate the corrections to the energies for the 1s and 2s levels in hydrogen that result from accounting for the fact that the proton is not a point charge.

b) Are states of higher angular momentum likely to be sensitive to the finite nuclear size? Explain and include plots to back up your argument.

c) Suppose that instead of an electron, a muon were bound to the proton. How would this affect the size of the energy corrections?