Quantum Mechanics II  
Midterm Exam #2

1. Two noninteracting, spinless particles, each with mass $m$, are placed in an infinite square well. Write down the wavefunctions and corresponding energies for the three lowest energy states for the following scenarios:

   a) The particles are distinguishable.

   a) The particles are identical bosons.

\[
\begin{align*}
\psi_{11} &= \frac{2}{a} \sin\left(\frac{\pi}{a} x_1\right) \sin\left(\frac{\pi}{a} x_2\right) \quad E_{11} = \frac{\pi^2 \hbar^2}{2ma^2} \\
\psi_{12} &= \frac{2}{a} \sin\left(\frac{\pi}{a} x_1\right) \sin\left(\frac{2\pi}{a} x_2\right) \quad E_{12} = \frac{3\pi^2 \hbar^2}{2ma^2} \\
\psi_{21} &= \frac{2}{a} \sin\left(\frac{2\pi}{a} x_1\right) \sin\left(\frac{\pi}{a} x_2\right) \quad E_{21} = \frac{\pi^2 \hbar^2}{2ma^2} \\
\psi_{11} &= \frac{2}{a} \sin\left(\frac{\pi}{a} x_1\right) \sin\left(\frac{\pi}{a} x_2\right) \quad E_{11} = \frac{\pi^2 \hbar^2}{2ma^2} \\
\psi_{12} &= \frac{\sqrt{2}}{a} \left[\sin\left(\frac{\pi}{a} x_1\right) \sin\left(\frac{2\pi}{a} x_2\right) + \sin\left(\frac{2\pi}{a} x_1\right) \sin\left(\frac{\pi}{a} x_2\right)\right] \quad E_{12} = \frac{3\pi^2 \hbar^2}{2ma^2} \\
\psi_{22} &= \frac{2}{a} \sin\left(\frac{2\pi}{a} x_1\right) \sin\left(\frac{2\pi}{a} x_2\right) \quad E_{22} = \frac{4\pi^2 \hbar^2}{ma^2}
\end{align*}
\]
2. A system is composed of two distinguishable spin 3/2 particles.

a) What are the possible values of the z-component of each of the two particles, and what are the possible values of the total spin of the system?

b) If the system is in a configuration in which the total spin is 3 and the z-component of the total spin is 2, then what are the possible values and corresponding probabilities that one could measure for the z-component of the spin of one of the individual particles?

Possibly useful: \( J_\pm |j, m\rangle = \hbar \sqrt{(j \mp m)(j \pm m + 1)} |j, m \pm 1\rangle \).

\begin{align*}
a) & \quad \text{Possible } S_{\text{total}}: 3, 2, 1, 0 \quad \text{Possible } M: \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2} \\
\Rightarrow & \quad |3, 3\rangle = |\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}\rangle \\
\Rightarrow & \quad \frac{1}{\sqrt{6}} |3, 3\rangle = \frac{1}{\sqrt{3}} |\frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{3}{2}\rangle + \frac{1}{\sqrt{3}} |\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{1}{2}\rangle \\
\Rightarrow & \quad |3, 2\rangle = \frac{1}{\sqrt{2}} |\frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{3}{2}\rangle + \frac{1}{\sqrt{2}} |\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{1}{2}\rangle \\
\text{Hence} & \quad 50\% \ \text{particle A has } M = \frac{1}{2} \ \& \ \text{particle B has } M = \frac{3}{2} \\
& \quad 50\% \ \text{particle A has } M = \frac{3}{2} \ \& \ \text{particle B has } M = \frac{1}{2} \\
\end{align*}
3. A particle is trapped in an infinite potential well of width $a$ that has a triangular bottom. The triangle is symmetric and centered at the middle of the well as shown in the figure below. The height of the triangular base is $V_0$. Use perturbation theory to calculate the ground state energy to first order.

Possibly useful: $\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$

$$E_o = \langle E_0 | \Delta H | E_o \rangle = 2 \int_0^{a/2} \gamma_o^* \Delta H \gamma_o \, dx = 2 \left( \frac{2}{a} \right) \left( \frac{2V_0}{a} \right) \int_0^{a/2} x \sin^2 \left( \frac{\pi x}{a} \right) \, dx$$

$$= \frac{8V_0}{a^2} \int_0^{a/2} x \sin \left( \frac{\pi x}{a} \right) \, dx = \frac{4V_0}{a^2} \int_0^{a/2} \left[ 1 - \cos \left( \frac{2\pi x}{a} \right) \right] \, dx$$

$$= \frac{4V_0}{a^2} \int_0^{a/2} x \, dx - \frac{4V_0}{a^2} \int_0^{a/2} \cos \left( \frac{2\pi x}{a} \right) \, dx = \frac{V_o}{2} - \frac{4V_0}{a^2} \left[ \frac{-a}{2\pi} \int_0^{a/2} \sin \left( \frac{2\pi x}{a} \right) \, dx + O \right]$$

$$= \frac{V_o}{2} + \frac{2V_0}{a^2} \int_0^{a/2} \sin \left( \frac{2\pi x}{a} \right) \, dx = 2V_0 \left( \frac{1}{4} + \frac{1}{\pi^2} \right) = E_o^1$$

$$E_o = \frac{\pi^2 \hbar^2}{2M a^2} \Rightarrow E_o \approx E_o^0 + E_o^1$$

Hence, $E_o \approx \frac{\pi^2 \hbar^2}{2M a^2} + 2V_0 \left( \frac{1}{4} + \frac{1}{\pi^2} \right)$
4. Write down the fundamental rules of quantum mechanics that we have discussed in class.

You’re damn right I’m asking this again!

<< See notes... I hope everyone knows these by now! >>