Basic Math for Physics Majors

No use of Mathematica is allowed for this assignment. You must present a logical, step by step derivation of your solutions to earn credit.

1. Simplify the following expressions as much as possible.

a)
\[
\frac{x(1 + x)}{x - 2} + \frac{18 - 36x}{6x^2 - 15x + 6}
\]

b)
\[
\frac{16 + (1 - \cos \theta)^4 + (1 + \cos \theta)^4}{2(1 - \cos \theta)^2 (1 + \cos \theta)^2}
\]

2. Solve the following equations for \(\theta\) and \(x\) respectively.

a) \(4 \sin^3 \theta = 2 \sin 2\theta + \sin \theta\)

b) \[
\frac{e^{-ax}}{1 + \beta e^{ax}} + \frac{6}{5} = \left[ 6 + e^{ax}(1 + \beta e^{ax}) \right]^{-1}
\]

3. Evaluate the following integrals.

a) \[
\int_0^8 (3x^4 - 5x^2 + 8)\delta(x - 1)dx
\]

b) \[
\int_{-5}^3 \ln(x + 2)\delta(x - \pi)dx
\]

c) \[
\int_{-2}^6 x^4 \left[ \frac{d^2}{dx^2} \delta(x - 3) \right] dx
\]
4. Let $z = a + ib$ for real constants $a$ and $b$. Determine the square modulus of the following complex numbers. Part (c) is understood to have parenthesis nested from the inside out, i.e., $\exp\left(i\exp\left(i\exp(i z)\right)\right)$ so that the exponents cannot be trivially carried down.

a) $\frac{3z + iz}{z - i}$

b) $\sin z$

c) $e^{i\exp i z}$

5. The gamma function $\Gamma(z)$ is a generalization of factorial to include real and even complex numbers. It is defined by

$$\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx.$$ 

Prove that $n! = \Gamma(n + 1)$. 